

Grappling with the denominator in the Western Cape Province

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1. Introduction

Accurate population estimates are important for appropriate planning, monitoring and evaluation of National and Provincial services. However, despite the completion of the three national censuses in 1996, 2001, and more recently in 2011, there remains uncertainty in the population estimates in the country, particularly at provincial and sub-provincial level. Various organisations and demographers have produced population projection estimates based on censuses and surveys with various assumptions incorporated in the projections on the current and future trends of mortality fertility and migration. While it is expected that population estimates may vary across sources, even those from the same source are inconsistent with each other.

At the outset, added to any error in the base estimate, it must be noted that all population projections have shortcomings and are only as good as the assumptions underlying the model of projection and the availability of calibration data. The greatest uncertainty in projections arises from difficulties in projecting levels of fertility trends, and over the longer term from mortality and migration. However, in the South African context the impact of HIV/AIDS has also contributed to the growing uncertainty of current and future mortality trends. Sub-population estimates are hindered by the non-availability of data and difficulty in estimating migration levels; the smaller the unit the more significant is migration which sometimes is even impossible to record due to continuous movement of populations in shorter time periods.

Inaccurate estimates may lead to inappropriate interpretation of data. Specifically, higher population estimates will indicate lower coverage of interventions and in the case of health, lower prevalence rates of disease. Therefore the following report compares the different population estimates available for the Western Cape. In addition, we consider the potential implications of the different estimates that are available for the 6 *health* districts of the Western Cape; Cape Town, Cape Winelands, Central Karoo, Eden, Overberg and the West Coast.

1.1 Data sources

Provincial population estimates have been obtained from several empirical and model based sources:

1.1.1 Empirical based estimates

Statistics South Africa:

- > 1996, 2001 and 2011 Census
- Community Survey 2007

1.1.2 Model based estimates

- Statistics South Africa mid-year estimates (available from 2002 to 2014)
- ASSA2008
- AltMYE CARe estimates
- PWC(Department of Social Development) (Price water house Coopers)
- PGWC Department of Health estimates (NZ)

There are a number of national population projections for South Africa done by UN agencies, the US Census Bureau and the Institute of Futures Research. They have not been included in this review as they do not have provincial break-downs.

2. Overview of underlying methodology in population estimates in South Africa and the Western Cape

2.1 1996, 2001 and 2011 Census

The census was an enumeration of the whole country carried out by Statistics South Africa in 1996, 2001 and most recently in 2011. The population estimates were adjusted for undercounts using adjustment factors from the post enumeration surveys (PES) undertaken shortly after the censuses (Statistics South Africa, 2012). The PES is used to evaluate the completeness of census coverage and content errors, that is the extent of overcount or undercount (Statistics South Africa, 2012). To evaluate the coverage of the 2011 Census, the PES was conducted in 600 enumeration areas (EAs) out of the total 100 576 EAs nationally and the adjustment factors were then derived from this.

2.2 2007 Community Survey

This is a nationally representative survey carried out by Statistics South Africa in February 2007 (Statistics South Africa, 2007). It was designed to provide data up to municipal level. The 2001 census enumeration areas were used as the sampling frame and about 247 000 households were selected using stratified cluster sampling strategy. The demographic estimates from the survey were adjusted to ensure coherence between different aspects of the survey as well as consistency with the 2001 census projected forwards taking into account the mortality, fertility and inter-provincial migration as measured from the 2007 community survey. In addition, the male population between the ages of 20 and 39 was adjusted for undercount using sex ratios to compensate for male undercount relative to females. These adjustments were made by province, population group, age and sex.

The ratio method (Shryock and Siegel 1976) of projecting geographic subdivisions was used to estimate the populations of the district councils and municipalities in the Community Survey, stratified by population group, sex and single-year-age. Most of the adjustments that were made to the municipality data showed that the direct measure by the Community Survey could not produce usable estimates in some of the municipalities with exception of densely populated municipalities like metros.

2.3 Statistics South Africa mid-year population estimates

Statistics South Africa, the national statistical office, publishes population estimates in the middle of each year. These are available nationally by population group, age and sex, and provincially by 5-year age groups and sex. Statistics South Africa has changed their methods of population projection over time, depending on the availability of new data, with at least three approaches used in the last 15 years. Statistics South Africa use the multi-regional approach developed by Willikens and Rogers (1978) and modified by the United Nations (1992) to estimate the provincial mid-year populations. This method basically requires the estimation of age-specific-migration rates (data rarely available) between each of the nine provinces. Migration rates were estimated form the 2011 census, derived from migration flows between provinces based on the migration questions asked in the census. The base provincial population by age and sex is from the 2001 census population, with the projections done up to 2014, with projection adjusted as to match the totals provincial population by sex from the 2011 census Stat(Statistics South Africa 2014).

Age-specific fertility rates were calculated from the recorded births (1998-2011) from vital registration, adjusted for late registration and completeness. Age-specific mortality rates by sex were calculated from the mortality and cause of death data sets (1998-2010) and life tables for males and females in the provinces were constructed using these age specific mortality rates. Then combining the migration, mortality and fertility rates the cohort component method was used to project the provincial population from the base year 2001.

Mid-year population estimates for the period 2002 and 2014 are available on the Statistics South Africa website, and the methodology given and explained above is documented for the 2013 mid-year estimates, and we assume same methodology was used for the 2014 mid-year estimates.

2.4 Actuarial Society of South Africa (ASSA2008 model)

These are Excel workbook models which project the South African population for each of the provinces by age, sex and population group. These models use the cohort component projection by individual ages. They use re-estimated populations in 1985 as a base population which produces reconstructed 1996 and 2001 census populations. This was derived by a process of reconstruction linking estimates of the population in 1970 to those of the census population in 1996 and 2001, ensuring consistency with estimates of fertility and mortality rates derived independently and between the numbers of males and females in various age groups (Dorrington, Johnson et al. 2005).

As the current provinces did not exist in 1985, they reconstructed the base population that could have been expected to be within these boundaries in that year. This was done by taking into account a remapping of the 1991 census into the new boundaries and the patterns of interprovincial migration between 1985 and 1996.

The births are estimated from the censuses and the 1998 Demographic Health survey (DHS) data and a reconstruction of the populations. Deaths are based on a wide range of sources up to 2008, which include the antenatal surveys, household surveys, antiretroviral treatment and prevention of mother-to-child transmission programs and registered deaths from vital registration. Internal migration is estimated from the censuses and reconstruction of the provincial populations. HIV/AIDS estimates are derived from the behavioural model of heterosexual transmission of HIV with survival since infection (Dorrington, Johnson et al. 2005).

2.5 AltMYE mid-year estimates (CARe)

The Centre for Actuarial Research at the University of Cape Town (CARe, UCT) produced an alternative set of mid-year estimates (AltMYE) with an age distribution that is consistent with that of the 2011 Census. The alternative mid-year estimates were produced by projecting the 2011 Census numbers annually backwards to 2001 and forwards to 2013 taking into account immigration and the migration between provinces as recorded in the 2011 census (Dorrington, 2013). (Statistics South Africa 2014)

2.6 PricewaterhouseCoopers (PWC) for the Department of Social Development

PricewaterhouseCoopers (PWC) prepared population projection estimates between 2011 and 2040 by age and sex at provincial, district and local district level (and sub-health district level for the Cape Town metro) for the Department of Social Development (PricewaterhouseCoopers 2014). They used the 2011 census by age and sex as the base population, fertility and mortality rates from the ASSA 2008 model and migration numbers at district and local district level from the 2011 census (PricewaterhouseCoopers 2014)

2.7 PGWC Department of Health

Assuming that the CARe projected estimates for the Western Cape Province between 2001 and 2013 are correct by five year age-groups and sex, and also assuming the Census 2001 and 2011 population estimates from the districts and sub-health districts by age and sex to be correct; the ratio method (Shryock and Siegel 1976) of projecting geographic subdivisions was used to estimate the populations of the district municipalities and sub-health district stratified by sex and five-year-age groups between 2001 and 2013.

3. Comparisons of the projections

3.1 The Western Cape Province population estimates



Figure 1: Comparison of the Western Cape population estimates

The ASSA 2008 estimates for total population are consistently lower than the StatsSA mid-year estimates and the AltMYE_CARe, with differences increasing over time (4% in 2002 compared to 11% in 2014).



Figure 2: Comparison of the Western Cape population estimates

Estimates of the age distributions of populations in the Western Cape Province from different sources are shown in figure 2. AltMYE_CARe is similar to Census 2011 as they use the census 2011 age distribution for their projections, whereas the StatsSA_MID2011 is completely different as it uses the 2001 age distribution in their projections.

The Census 2011 age distribution has a different age structure to the earlier 2001 Census and the 2007 Community survey as well as the later projections by Statistics South Africa of 2014 (StatsSA_MID2014). This is due to the effect of migration in the Province, with the rate of in-migration into the province increasing over time especially in the middle age groups (20-40).

Most of the differences in the age distributions are for the population below the age of 35. This is a huge concern to planners in general as the population under the age of 35 is about two thirds of the overall provincial population irrespective of the source. This means that the population estimates below the age of 35 are either being under or over-estimated depending on which population source that is used, this in turn leads to rates that depend on population by age as a denominator being either over or under-estimated.

3.2 Cape Town population estimates



Figure 3: Comparison of the Cape Town population estimates

There are small differences between the total populations' estimates in the Cape Town metro, with differences narrowing over time from the different sources. The seemingly large differences between sources in 2001 are due to the correction for under-enumeration that was made in the projected population estimates, which was not corrected for in the 2011 Census. The 2007 Community survey falls in between the two projection estimates, also a

difference in the adjustment factors used for the BOD projections and the StatsSA mid-year estimates.



Figure 4: Comparison of Cape Town population estimates: males

Similar differences and patterns in age distributions can be seen in the Cape Town population for both males and females as was seen in the Western Cape Province, with most of the differences in the age distributions especially below the age of 40 (**Figure 4 and Figure 5**). BOD_2013, PWC_2013 and Census_2011 have identical age structures which is different from the age distribution in the StatsSA_mid-year 2013 estimates. Note that the StatsSA_mid-year 2013 follows the same age structure from the 2001 Census, showing that Statistics South Africa are using the 2001 age structure in the projection of their mid-year estimates, thereby not accounting for the migration being noted in the young adults between the ages of 20 and 40 in both males and females. The migration can be seen in the CS_2007 age distribution between the ages of 15 and 30, which dramatically increased as seen in the 2011 Census between the ages of 20 and 40 in both males and females.



Figure 5: Comparison of Cape Town population estimates: females

3.2.1 Sub-health districts, Cape Town



Total population by sub-health district in Cape Town, 2011

Figure 6: Comparison of the population in the new Sub-health districts of Cape Town, 2011

Population estimates in the health districts of Cape Town for 2011 derived from Census 2011, BOD 2011 and StatsSA mid-year 2011 are shown in figure 6. The total populations are similar in all the health districts except in Khayelitsha, Mitchells Plain and Southern. These differences are most probably due to the boundary demarcations that are being used by StatsSA and those that are being used locally. For the 2011 Census and BOD estimates boundaries were obtained from the city of Cape Town, where it was noted that there are some suburbs in the census that cross cut between sub-health districts, especially between Mitchells Plain, Klipfontein and Southern, hence the city re-distribute the populations in these suburbs according to their boundaries, of which StatsSA do not have this knowledge presumably, hence they allocate whole suburbs into the wrong health districts



Total population by sub-health district in Cape Town, 2013

Figure 7: Comparison of the population in the new sub-health districts of Cape Town, 2013

Note that from figure 7 the small differences between BOD and the StatsSA mid-year estimates seen in 2011 increase in 2013, as there is a ripple effect of the differences as the population is projected, thus the further the projection years the bigger the differences become over time.

3.2.2 Health districts in Cape Town over time

Figure 8 shows population estimates by health district over time for BOD and StatsSA mid-year estimates. The BOD and StatsSA projections are similar in the Northern, Tygerberg and Western sub-health districts over time. While for Khayelitsha and Southern the estimates initially start of similar, they start deviating away from each other after 2007, with StatsSA estimates greater than the BOD estimates. For Eastern StatsSA estimates are greater than BOD estimates and they cross around 2011, while for Klipfontein BOD estimates are greater than StatsSA estimates and also cross in 2011. Mitchells Plain has the biggest differences in the populations between BOD and Stats SA, with a consistent difference over the time period 2002 to 2013, with BOD estimate greater than the StatsSA estimates. The differences between the BOD and StatsSA estimates vary in each of the sub-health districts, with differences either increasing or decreasing over time. Most of these differences can be attributed to the mis-allocation of population in the sub-health districts, with StatsSA using different boundaries than those that are used locally by the city of Cape Town. Some of the differences could also be due to the different assumptions that have been used in the projection of these estimates





500000 400000 . 300000 200000 100000 2016-2017-2018-2010-2012-2013-2014-2015-2002 2008 2009 2011 2001 2003 2007 2004 - BOD ---- StatsSA .

Khayelitsha sub-health district, total population

600000









Figure 8: Comparison of the sub-health districts of Cape Town, BOD and StatsSA total populations over time

4. Discussion

A population grows or declines through the interaction of three factors; namely fertility, mortality, and migration. To project population size, assumptions about how these factors change over time are essential, this adds to the complexity due to uncertainty of future trends of fertility, mortality, and migration and their relationship with each other change over time. Unfortunately different sources, in addition to using different base populations and projection methods, have made different assumptions around these factors, resulting in population estimates that are inconsistent with each other.

The techniques and methods used do not seem to guarantee any accuracy in the population estimates but more important is the input data and the assumptions made. The complex methods require a large pool of data and more detailed assumptions. In the absence of these requirements there is more uncertainty around the projection estimates. All these factors regrettably present a lot of uncertainty in all these population estimates which makes it imperative for the users to understand the reliability and the limitations of the projected populations. Awareness of the effect of using the different estimates in the projected population needs to be identified and explained in detail.

All the sources in this overview show that the population of the Western Cape is growing over time, but at different rates. The differences in the projected population estimates from these different sources are widening over time.

The longer the projection period the greater the errors to be found in these population estimates due to the compounding effects of incorrect assumptions over time. It is better to project over shorter time periods than longer time periods. Thus for shorter time periods the major source of errors is in the population data at the start of the projection, whilst for longer intervals assumptions about future fertility, mortality, and migration trends have a greater effect on the accuracy. So at this point it would be better to select a population projection that has a base population that is nearer to the current period or similar in size to the population currently. This would minimise the errors in the population estimates, for example by selecting the Stats SA projection with a base in 2001 over the ASSA 2008 projection which has a base in 1985.

Although the accuracy of these estimates can be improved by annual updates of any changes in the population if they are available, this rarely happens. The selection of these population trends has clear repercussions on monitoring trends in mortality rates or coverage rates such as immunisation and TB incidence.

There are more estimates available on a provincial basis. The Western Cape as a larger area has more population estimates than the sub-health districts within the province. This is due to less uncertainty and assumptions that are made at larger areas than in smaller areas. As shown in this overview, the Western Cape Province has 7 different population estimates, although some are from the same organisations but at different times, whereas at the sub-districts, only have two sets of estimates from the BOD and StatsSA. The BOD and StatsSA population estimates vary within the sub-districts in Cape Town with contrasting trends over time as well (Figure 8). The biggest concern is the huge discrepancies in age distributions that are being used, especially below the age of 40 at provincial and sub-provincial level. What is also of concern are the official figures from StatsSA, where the age structure from the 2011 census is not being used in the mid-year population estimates, as they continue to stick to the 2001 age structure. This in turn has a huge impact on targeted interventions for specific age groups below the age of 40, as this is grossly underestimating the effect of migration.

Though censuses serve as the empirical foundations for the population on which projections are based, with vital statistics, surveys and immigration data serving as the empirical foundation for births, deaths and immigrants, sometimes the accuracy of these sources is also questioned, it is the users' prerogative to decide which projection they would feel comfortable with to produce mortality and health statistics.

5. Recommendations

There is uncertainty about the exact population in the province with alternative estimates and projections giving different numbers and no clarity on the definitive population projection. It is strongly recommended that:

- There is a re-estimation of the projected population, incorporating the age structure from the 2011 census, and back projecting for earlier years to obtain a consistent series that can be used for monitoring trends.
- Assumptions made and methods used in projections are clearly stated and explicitly documented so that other users might be able to reproduce the population estimates.
- There is increased transparency in the population estimates by providing easy access to the data and methods.
- There should be harmony in the estimates being produced by the different organisations. This can be achieved by forming an advisory board that would bring together and allow population experts from different organisations to provide input and knowledge on the best possible way of coming up with projection estimates to ensure that the projections are being done in a plausible and acceptable manner.
- Furthermore the projections should be published along with an analysis and explanation of changes since the last projection, that is, the new projections should be compared with past projections noting any changes that have been made and changes that are apparent.
- Projections from the various bodies should report on the main errors in recent and past projections and provide some evaluation of the quality of their estimates.
- More effort needs to be put into making small area population estimates available and more accessible, with a review of the techniques currently used to project these estimates.
- StatsSA liaise with the City of Cape Town so as to use same boundaries for the sub-health districts in the metro when they produce their mid-year estimates as they are the official body producing official statistics, hence they need to

produce estimates that local people will be happy to use in their areas of interest.

Statistics South Africa to use the 2011 Census age structure in their mid-year estimates. The Centre for Actuarial Research (CARe) state that there is mounting evidence that the age distribution of the census 2011 is probably closer to the truth than that of the official mid-year estimates produced by Statistics South Africa, and is also largely consistent with the age distribution of the 2001 Census. Stats SA's asserts that "neither source is more right than the other; (because) they serve different purposes" (Press Statement, 17 July 2013), yet these differences have the potential to confuse, researchers, demographers and planners as to which numbers to use for planning, monitoring, and weighting data from surveys.

In the meanwhile, although government has to use official statistics, which may result in inconsistencies, it is extremely important for the Western Cape Department of Health(even the province as a whole), which aims to monitor trends to use a consistent set of estimates to examine trends in different rates and ratios

In the longer term, Statistics South Africa is urged to make every effort in collecting accurate and precise information during the next census and survey enumerations, so as to make sure reliable estimates on which projection estimates can be based upon are collected. This will have a huge impact on projections, as this will result in less assumptions and corrections to be made in the data and much simpler methods to be used if sufficient and more accurate data are collected.

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